1)

a) **Derive C -> B**

First we can see that A -> BC which according to decomposition gives us A -> B & A -> C. After that we also see that C -> A which according to transitivity with A -> B gives us C -> B

b) **AE -> F**

Using DE -> F and pseudo-transitivity we can see that if we find a A such that A -> D we get AE -> F.  
We know that A->C and C->D, using transitivity we get A -> D which in turn lets us derive DE -> F => AE -> F

2)

a) **Attribute closure of set: X = { A }**

FD1: A -> BC **=>** A -> B & A -> C

X+ = { A, B, C }

FD2: C -> AD **=>** A -> AD which means A -> D

X+ = { A, B, C, D }

FD3: DE -> F **=>** If A -> D then AE -> F which means A -> F

X+ = { A, B, C, D, F }

b) **Attribute closure of set**: X = { C, E }

FD1: A -> BC **=>** Because C -> A then C -> B

X+ = { C, E, B }

FD2: C -> AD **=>** C -> A & C -> D

X+ = { C, E, B A, D}

FD3: DE -> F **=>** C -> D gives us CE -> F

X+ = { C, E, B, A, D, F }

3)

a) **Determine the candidate key(s) for R(A, B, C, D, E, F)**

A is the only value not in RHS. We look at {A}+:

FD1: AB -> CDEF  
FD2: E -> F  
FD3: D -> B

{A}+ = {A, C, D, E, F, B} which means it is a super key and thus the only CK

b) Note that R is not in BCNF. Which FD(s) violate BCNF condition?

FD2 and FD3 violates BCNF.

3) a) **Determine the candidate key(s) for R(A, B, C, D, E, F)**

**Ans:**

FD1: AB -> CDEF  
FD2: E -> F  
FD3: D -> B

Candidate keys for R:

A is not in RHS; must be present in Ck.

F and C are not in LHS; can not be present in a Ck.

X = {A,B},

X+ = {A,B,C,D,E,F} FD1:AB ->CDEF

X+ = {A,D,B,C,E,F} FD2: E->F

X+ = {A,D,B,C,E,F} FD2: D->B

X = {A,D}

X+ = {A,D,B,C,E,F} FD1:AB ->CDEF

X+ = {A,D,B} FD3: D -> B

X = {A,E}

X+ = {A,E,F} FD2:E->F

Candidate key’s are {A,B} and {A,E}

b) Note that R is not in BCNF. Which FD(s) violate the BCNF condition?

**Ans**: FD2 and FD3 do not contain a superkey. Hence, they violate the Boyce Codd Normal Form.

c) Decompose R into a set of BCNF relations, and describe the process step by step (don't forget to determine the FDs and the candidate key(s) for all of the relation schemas along the way).

**Ans:**

Decompose based on FD3: E->F (as it violates BCNF):

R1: (F,E) with all attributes in X and Y

R2: (A,B,C,D,E) with all attributes in R, without the ones in Y and not in X.

FD4: AB -> CDE ; FD1: AB -> CDEF

FD5: AD -> BCE ; FD1: AB -> CDEF

Decompose based on FD2: D->B (as it violates BCNF)

R1: (D,B) ; FD3 : D-> B ; candidate key: {D}

R2: (A,C,D,E) ; FD9 : AD -> CE; candidate key: {AD}

FD6 -> AD -> AB ; FD3 augmented

FD7: AD -> CDEF ; FD1 and FD4 transitivity

FD8: AD -> CDE; Decomposition of FD7

FD9: AD -> CE; Decomposition of FD8

4) Consider the relation schema R(A, B, C, D, E) with the following FDs

FD1: ABC → DE

FD2: BCD → AE

FD3: C → D

a) Show that R is not in BCNF.

**Ans:** FD3 violates BCNF, C is not a superkey.

The elements that are not in LHS but are in RHS:

X+ = {B,C}

X+ = {B,C,D} ; FD1: C -> D

X+ = {B,C,D,A,E} ; FD2 : BCD -> AE

Candidate key is {B,C}

b) Decompose R into a set of BCNF relations (describe the process step by step).

**Ans:**

R1: {C,D} ; FD3: C-> D ; ck = {C}

R2: {A,B,C,E} ;

FD4: C -> CD

FD5: BC -> BCD (Augmentation of FD4)

FD6: BC -> AE (transitivity of FD5)

Decomposing:

BC -> E ; (BC is a candidate key)